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GAMMA-RAY EMISSION FROM DARK MATTER WAKES OF RECOILED BLACK HOLES

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ABSTRACT

A new scenario for the emission of high-energy gamma-rays from dark matter annihilation around massive black holes is presented. A black hole can leave its parent halo, by means of gravitational radiation recoil, in a merger event or in the asymmetric collapse of its progenitor star. A recoiled black hole which moves on an almost radial orbit outside the virial radius of its central halo, in the cold dark matter background, reaches its apapsis in a finite time. Near or at the apapsis passage, a high-density wake extending over a large radius of influence, forms around the black hole. Significant gamma-ray emission can result from the enhancement of neutralino annihilation in these wakes. At its apapsis passage, a black hole produces a flash of high-energy gamma-rays whose duration is determined by the mass of the black hole and the redshift at which it is ejected. The ensemble of such black holes in the Hubble volume is shown to produce a diffuse high-energy gamma-ray background whose magnitude is compared to the diffuse emission from dark matter haloes alone.

Subject headings: cosmology: dark matter

1. INTRODUCTION

We study the effect on dark matter (DM) distribution of black holes (BHs) that are ejected from their parent haloes but remain on bound orbits outside the virial radius. Black holes can be recoiled due to anisotropic emission of gravitational radiation (Bekenstein 1973, Fitchett 1983, Merritt et al 2004). As an ejected black hole (BH) moves in the dark matter-dominated universe, a *wake* forms whose density is maximum downstream along the symmetry axis. The wake density and its extent, *i.e.* the zone of influence of the BH, increase with decreasing velocity of the BH and velocity dispersion of the DM environment. A black hole on bound orbit round its parent halo comes to rest at apapsis of its highly radial orbit in a finite time. In a cold dark matter background with a velocity dispersion of the order of cm/s, a BH near its apapsis has a large radius of influence. If DM were to consist of self-annihilating particles such as neutralinos, significant γ-rays would be emitted from these wakes. On its apapsis passage, a BH is shown to produce a flash of high energy γ-rays whose duration depends on the mass of the BH and the redshift at which it is ejected. We also estimate the total isotropic diffuse gamma ray background emitted from the ensemble of cosmological wakes in the Hubble volume, assuming that only a small fraction of BHs leave their haloes and compare it to the background from DM haloes alone.

There are two main assumptions made in this work. First, we assume that outside haloes, the cold dark matter environment is approximately homogeneous: an assumption which is more valid at high redshifts. Second, to evaluate the wake density we assume a constant-velocity approximation, whose validity is questionable and remains to be confirmed by future N-body simulations.

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2. DENSITY OF THE WAKE

The expression for the density enhancement due to a moving point mass in a thermal environment is (Danby & Camm 1957, Griest 1988)

$$\frac{\rho}{\bar{\rho}}=\int_{u=0}^{\infty}du\,\frac{u\sqrt{u^2+q^2}}{(2\pi)^{3/2}}\int_{\lambda=0}^{\pi}\sin\lambda\,d\lambda\int_{\nu=0}^{2\pi}d\nu\,e^{-F/2} \qquad (1)$$

where $F = p^2 + u^2 + 2pu(uZ + q^2\cos\theta/2 - Z\sqrt{u^2+q^2}\cos\lambda)/(u^2 + q^2/2 - u\cos\lambda\sqrt{u^2+q^2})$, $p = V_{\rm BH}/\sigma_{\rm DM}$, $q = 2GM_{\rm BH}/r/\sigma_{\rm DM}^2$, $Z = \sqrt{u^2+q^2}(\cos\theta\cos\lambda - \sin\theta\sin\lambda\sin\nu)$, $\bar{\rho}$ is the density of the environment of the BH, $M_{\rm BH}$ is the mass of the BH moving with velocity $V_{\rm BH}$ and the distance r and angle θ are the radial distance measured from the BH position and the angular position is measured from the symmetry axis with $\theta=0$ upstream in front of the BH and $\theta=\pi$ downstream on the symmetry axis behind the BH.

When the velocity dispersion of the medium is very low in the limit $V_{\rm BH} >> \sigma_{\rm DM}$, the integral in (1) becomes highly oscillatory and difficult to evaluate. The expression for the density enhancement in this limit ($V_{\rm BH} >> \sigma_{\rm DM}$) is (see Sweatman & Heggie 2004 for details, also Sikivie & Wick 2002 for a different approach)

$$\frac{\rho}{\bar{\rho}}=\frac{2\,V_{\rm BH}^2}{\pi\sigma_{\rm DM}^2}\int_0^{\infty}\!\!\int_{\alpha=0}^{\pi}n\,dn\,d\alpha\,e^{-V_{\rm BH}^2n^2/\sigma_{\rm DM}^2}\frac{f}{\sqrt{f^2-1}} \qquad (2)$$

where $f = 1 + (rV_{\rm BH}^2)/(2GM_{\rm BH}) \times (1 + \cos\theta - 2\sqrt{1+\cos\theta}\,n\cos\alpha + n^2)$. the density (2) along the symmetry axis ($\theta=\pi$) attains the maximum value: $\rho_{|axis} \approx \frac{\bar{\rho}(z)}{\sigma_{\rm DM}(z)}\sqrt{\pi\,G\,M_{\rm BH}/r}$ for $\theta=\pi$ and $\sigma_{\rm DM}^2 << GM_{\rm BH}/r$. The wake density is independent of the velocity of the BH, downstream along the symmetry axis due to the finite velocity dispersion of dark matter.

3. DENSITY ENHANCEMENT AND RADIUS OF INFLUENCE: THE RÔLE OF BH VELOCITY

The zone of influence of BH decreases with increasing velocity and velocity dispersion of its environment as

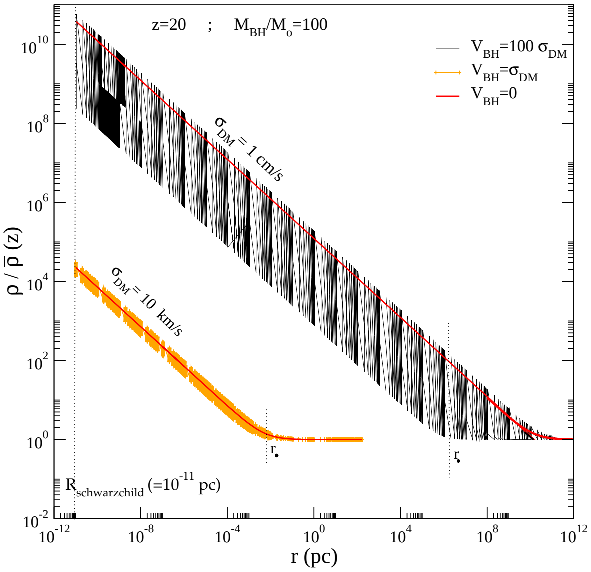

FIG. 1.— The change in the wake overdensity (1) with radial distance r from BH in a hot environment (lower curve) and cold environment (upper curve). The lower and upper envelopes to the black bands correspond to $\theta = 0$ and $\theta = \pi$ respectively. The solid (red) curves are for stationary BHs (3).

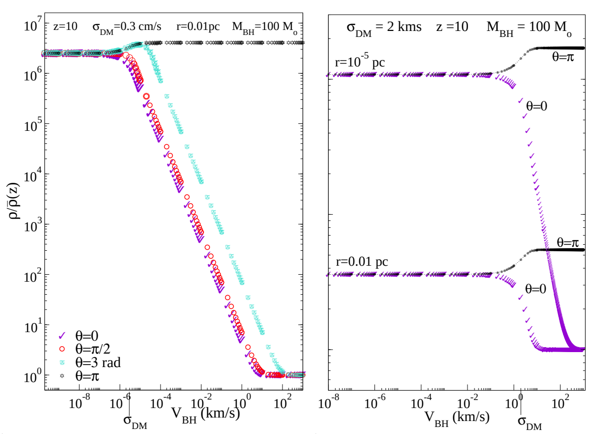

FIG. 2.— The density (1) for $V_{\rm BH} \leq \sigma_{\rm DM}$ and (2) for $V_{\rm BH} >> \sigma_{\rm DM}$ are shown for a BH moving in a cold ($\sigma_{\rm DM} = 0.03$cm/s) [left panel] and hot ($\sigma_{\rm DM} = 2$km/s) [right panel] environment.

shown in Figs. 1 and 2. In Fig. 1, produced by numerical integration of equation (1), the red middle curve for $V_{\rm BH} = 0$ shows that this approximation provides a good description of the average wake density of slowly-moving BHs. In the limit as $V \to 0$, the density profile of the wake (1) reduces to

$$\frac{\rho}{\bar{\rho}} = \sqrt{\frac{4}{\pi}}\sqrt{\frac{r_\bullet}{r}} + e^{r_\bullet/r}\mathrm{Erfc}\left(\sqrt{\frac{r_\bullet}{r}}\right), \qquad (3)$$

where Erfc is the complementary error function and $r_\bullet$ is the radius of influence of the BH: $r_\bullet = GM_{\rm BH}/\sigma^2_{\rm DM}$. We emphasis that (3) is the limit $V \to 0$ of (1), and is not a unique density profile for stationary BHs. Here, we use (3) only as an approximation to (1) for slowly-moving BHs. Fig. 2, shows the dependence of the density enhancement on the BH velocity and DM velocity dispersion. When the BH is moving fast with respect to the background, a significant density enhancement only arises in a small zone around the symmetry axis (downstream) of the BH. The density enhancement also decreases with increasing velocity dispersion of dark matter environment. The highest density enhancement and largest radius of influence are achieved for BHs moving slowly in a cold background.

4. DRAG FORCES ON THE EJECTED BH: TIME TAKEN TO REACH THE APAPSIS

The BH remains bound to its central halo if it is ejected with a velocity less than the escape velocity (measured from the virial radius). Because it is ejected from the centre (and also when with a large velocity), the BH is on almost radial orbit. The BH initial velocity is set as follows. We assume that the halo mass is about 2×10^4 times the mass of the BH (Madau & Rees 2001). Thus, for a BH of mass $M_{\rm BH}$, the virial radius of the halo, from which it was ejected, can be determined using $M_{\rm halo} = 4\pi/3\Delta_{\rm vir}(z)\bar{\rho}(z)R^3_{\rm vir}(z)$ and noting that $\Delta_{\rm vir}(z) = (18\pi^2 + 82x - 39x^2)/\Omega(z)$ and $x = \Omega(z) - 1$ and $\Omega(z) = \Omega_m(1+z)^3/[\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2]$ (see Bullock et al 2001 for details). Having evaluated the virial radius, we can then evaluate the escape velocity from the virial radius of the halo, using $V_{\rm escape}(z, M) = \sqrt{2GM_{\rm halo}/R_{\rm vir}}$.

The ejected BH is slowed down by the gravitational pull of its parent halo and also by the dynamical friction of dark matter background as

$$\frac{dV_{\rm BH}}{dt} = -\left[\frac{(2\,E + V^2_{\rm BH})^2}{4\,G\,M_{\rm halo}} + \frac{4\,\pi\,G^2\,M_{\rm BH}\bar{\rho}\ln(\Lambda)}{V^2_{\rm BH}}\right] \qquad (4)$$

where $E = -V_i^2/2 + G\,M_{\rm halo}/R_{\rm vir}$ is the absolute value of the energy with which a bound BH leaves the virial radius with velocity V_i. Since dynamical friction plays a sub-dominant rôle in *braking* the BH, the values of $\ln(\Lambda)$ and the background density $\bar{\rho}$ marginally affect the value of (4) for a BH in its initial outward journey. We have set $\ln(\Lambda)$ to unity and $\bar{\rho}$ to $\bar{\rho}_0$ in order to obtain an upper value on the time to apapsis. The left panel of Fig. 3, produced from numerical integration of equation (4) demonstrates that the time elapsed, $t_* - t_i$, since the BH was ejected till it reaches the apapsis, is far shorter than the Hubble time, t_0, as long as the BH escapes the virial radius with a velocity less than the escape velocity. This figure also demonstrates that $t_* - t_i$ is independent of the BH mass, because higher mass BHs need larger velocities to leave their more massive haloes.

5. GAMMA-RAY FLASHES FROM BHS AROUND APAPSES

Cold dark matter if composed of neutralinos would self-annihilate into secondary products, including energetic photons (e.g. see Bertone, Hooper, Silk 2005 for a recent review). The *absolute luminosity*, in units of $\gamma{\rm s}^{-1}$ of a BH of mass M at redshift z is:

$$L(M, z) = \left[\mathcal{N}_\gamma \frac{\langle\sigma v\rangle}{2m^2_\chi}\right]\left[4\,\pi\int_{r_s}^{r_\bullet} r^2 dr\,(\rho(M, z, r))^2\right] \qquad (5)$$

where r_s is the Schwarzchild radius, $r_\bullet$ is the radius of influence of the BH, m_χ is the neutralino mass (~ 100 Gev/c^2), $\langle\sigma v\rangle$ is the interaction cross-section [which we fix at 2×10^{-26} cm^3/s] and $\mathcal{N}_\gamma$ is the number of photons produced per annihilation. Note that the integral (5) is independent of angle θ for stationary BHs.

We had found that the wake density of a slowly moving BH is well-approximated by the wake density of a

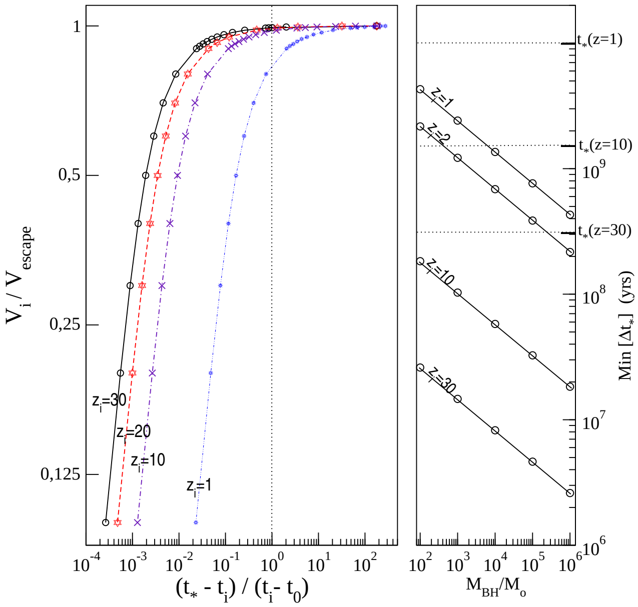

FIG. 3.— *Left panel:* The ratio of initial BH velocity at the virial radius, V_i, to escape velocity, versus ratio of time of ejection of BH, t_i, to when it reaches apapsis, t_*, to the time left from the moment of ejection to now $(t_0 - t_*)$. *Right panel:* Minimum time the BH spends around the apapsis when its luminosity dominates over that of its parent halo [using expression (10) in (9)]. The total time, t_*, taken to reach the apapsis from the virial radius is also shown by the dotted horizontal lines for different redshifts, for BHs ejected with near escape velocity.

stationary BH. By inserting (3) [keeping only the first term] in (5) we obtain the analytic expression (in units of $\gamma\, s^{-1}$)

$$L_{\rm BH} = 1.4\ \times\ 10^{25}\,(1+z)^4\left(\frac{M_{\rm BH}}{M_\odot}\right)\,R^2_{\rm cutoff} \qquad (6)$$

for the absolute luminosity of a BH, where $R_{\rm cutoff}$ is in unit of parsecs. In (6) $R_{\rm cutoff}$ is the radius within which the luminosity of the BH is evaluated.

Next, the BH luminosity is compared to the absolute luminosity of its central dark matter halo of mass $2 \times 10^4\, M_{\rm BH}$, assuming it has a NFW density profile (Navarro, Frenk & White 1997)

$$\frac{\rho}{\bar{\rho}_0} = (1+z)^3\frac{\Omega_m}{\Omega(z)}\frac{\delta_c}{[(c\,r/R_{\rm vir})(1+(c\,r/R_{\rm vir}))^2]} \qquad (7)$$

where $\Omega(z) = \Omega_m(1+z)^3/[\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2]$, $\delta_c = (\Delta_{\rm vir}/3)c^3/(\ln(1+c) - c/(1+c))$, and c is the concentration parameter, for which we use the fit $[10/(1+z)](M_{\rm vir}/M)^{-0.13}$ (agrees well with Bullock et al 2001, Hennawi et al 2007) and $M_{\rm halo} = M_{\rm vir}$ is the halo mass within the virial radius. The absolute luminosity L (5), in unit of $\gamma\, s^{-1}$, of a NFW halo of mass $M_{\rm halo}$ can then be evaluated using (7) in (5) [after setting $M = M_{\rm halo}, r_\bullet = R_{\rm vir}, r_s = 0$] and fitted by the following functions

$$L_{\rm NFW}(z>1) = \frac{5.6\times10^{27}}{(1+z)^{-1/3}}\left(\frac{M_{\rm halo}}{M_\odot}\right)^{0.7\,(1+z)^{0.075}}$$
$$L_{\rm NFW}(z\le1) = \frac{1.6\times10^{29}}{(1+z)^{3}}\left(\frac{M_{\rm halo}}{M_\odot}\right)^{0.7} \qquad (8)$$

We can find an analytic expression for $R_{\rm cutoff}$ by using the dynamical friction formula (4) and assuming that near the apapsis dynamical friction dominates over the gravitational pull of the halo. This yields

$$R_{\rm cutoff} = \frac{\sigma_0^4(1+z)\alpha^4}{16\pi G^2 M_{\rm BH}\bar{\rho}_0}\ ;\ \Delta t_* = \frac{\alpha^3\sigma_0^3}{12\pi G^2 M_{\rm BH}\bar{\rho}_0} \qquad (9)$$

where Δt_* and $R_{\rm cutoff}$ are defined as the time-duration and distance-parcours during which the velocity of BH reduces from a fraction α of the background velocity dispersion to zero (at the apapsis), i.e. $0 < V_{\rm BH} < \alpha\sigma(z)$ where $\sigma(z) = (1+z)\sigma_0$ and σ_0 is present velocity dispersion of DM (approximately 0.03 cm/s for neutralinos). The lower-bound on $R_{\rm cutoff}$ and Δt_* is obtained by requiring that $L_{\rm BH}/L_{\rm NFW} > 1$ where $L_{\rm BH}$ is given by (6) and $L_{\rm NFW}$ is given by (8). These lower bounds are shown in Fig. 4 and Fig. 3 respectively. Fig 4 shows that at high redshifts most ejected BHs are more luminous than their parent haloes whereas at low redshifts a much higher $R_{\rm cutoff}$ is required. Similarly, we put a lower limit on the parameters α

$$\alpha > \frac{7\times10^4}{(1+z)^{0.7}}\left(\frac{M_{\rm BH}}{M_\odot}\right)^{1/4} \qquad (10)$$

for $z > 1$ and for $z \le 1$ we can use the very similar expression $\alpha > (10^5/(1+z)^{9/8})(M_{\rm BH}/M_\odot)^{1/4}$ which are shown in Fig. 4. The two panels of this figure show that less massive BHs at higher redshifts are the most luminous.

6. DIFFUSE GAMMA-RAY BACKGROUND

In a cosmological scenario, ejected BHs near their apapses passages, especially those at high redshifts, where the merger rate is higher, can yield an observable diffused background flux. The total flux is given by the integral

$$\Phi = \int_M\int_z \frac{L(M,z)}{4\pi r(z)^2}\,N(M,z)\,dM\,d\mathcal{V}(z) \qquad (11)$$

where M can be either the BH mass ($M_{\rm BH}$) or the halo mass ($M_{\rm halo}$), $r(z) = R_H\left(1 - \frac{1}{\sqrt{1+z}}\right)$ with Hubble radius $R_H = 4000$Mpc, and $N(M,z)$ is the number density of the BHs [or haloes in the calculation for NFW haloes] and the luminosity of a single BH $L(M,z)$ (or the parent NFW haloes at z) is given by (6) [or (8)] and the volume element is $d\mathcal{V} = \sin\psi\, r(z)^2 dr(z) d\psi\ d\varphi$.

The physical number density of the BHs is assumed to follow the Press-Schechter formalism (Press & Schechter 1974, Bower 1991), multiplied by the *relative* time a BH spends at apapsis, *i.e.* $(\Delta t_*)/t_0$ where t_0 is the age of the Universe.

The time spent at apapsis is itself a function of BH mass as given by (9) and shown in the right panel of Fig. 3 for minimum value of α [obtained by substituting (10) in (9)]. Thus only the fraction $\Delta t_*/t_0$ of the ejected BHs can be considered to be actually on their apapses passage in a Hubble time. We can evaluate the total flux by performing the integrals in (11) [multiplied by Min$[\Delta t_*]/t_0$ for BHs]. For spectral index $n = -1$ and $M_* \sim 10^{12}\, M_\odot$ and for the ensemble of BHs at their apapses passages and their central haloes (assuming NFW profiles) we obtain $\Phi_{\rm NFW} \sim 10^{-6}\quad \gamma\ \ {\rm cm}^{-2}{\rm sr}^{-1}$ The flux from the BHs is lower than this value (since we have evaluated our parameters $R_{\rm cutoff}$ and α by requiring $L_{\rm NFW} = L_{\rm BH}$) because the BHs only spend a fraction

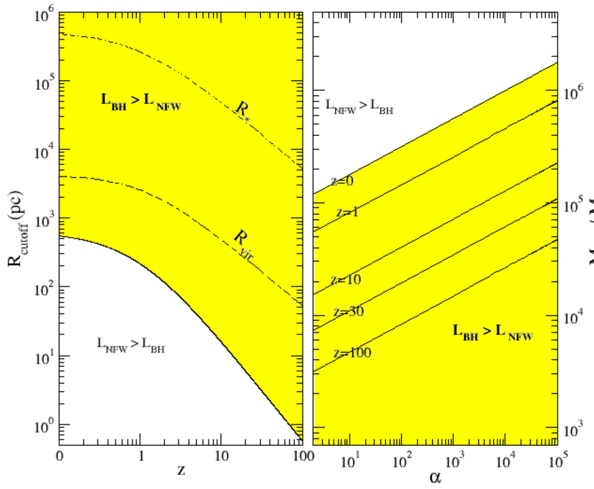

FIG. 4.— *left panel:* The minimum radius $R_{\rm cutoff}$ at apapsis required for the BH luminosity to dominate over that of its parent halo is shown by the lower solid line. The minimum virial radius, $R_{\rm vir}$, of the halo (corresponding to ejection of a BH of mass $100M_\odot$) and similarly the minimum distance from the virial radius of the apapsis R_* are also plotted. *right panel:* the minimum value of the parameter α as a function of redshift and mass of the ejected BH [expression (10)] are shown.

of time at the apapsis [Min(Δt_*)] which yields approximately $\Phi_{\rm NFW} \sim 10^{-14}$ γ cm^{-2}sr^{-1} The flux would be attenuated due to interaction of photons which however would affect *approximately* equally $\Phi_{\rm BH}$ and $\Phi_{\rm NFW}$. We have assumed that only BHs produced in 3σ peaks of the density perturbation can undergo effective mergers. We have assumed that all ejected BHs orbit their central haloes outside the virial radius; however, were this not the case, we do not expect any significant overall decrease in the flux which is already underestimated by our moderate choices of parameters and also by assuming that there is only one apapsis passage for a BH. We have also ignored the effect of multiple density-enhancement for a BH which is on its inward journey through an already high-density wake.

In conclusion, BHs on bound orbits around haloes can be powerful sources of high-energy γ-rays, both individually as resolved sources and collectively as diffuse background. The results here indicate that the globular clusters in the outskirt of our halo or field galaxies in our local Universe devoid of central BHs can have orbiting BHs which during their apapsis passages would produce flashes of high-energy γ-rays, although this effect is expected to be most significant at high redshifts. The validity of dynamical friction formulae has been very rarely studied for radial orbits (Gualandris & Merritt 2007). The fact that there is no mass-loss makes BHs a rare case for dynamical friction theory. Throughout this work we have assumed a homogeneous background and a constant-velocity approximation, both of these assumptions are questionable for the problem considered here. It thus remains for forth-coming numerical works to check the extent of the validity of the present results.

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